that for large x . Since

$$
F_n(x) = g^n x^{n(\lambda - 2) + 1} (\ln x)^n r^{2n} \left[\Gamma(n+1+\nu) n! \right]^{-1}
$$

× $\left[1 + O((\ln x)^{-1}) \right].$

Thus, in the same sence as (3.16) , we get immediately that

$$
F(x) \sim \Gamma(1+\nu)x(\frac{1}{2}z)^{-\nu}I_{\nu}(z)
$$
, \t(E16) $B_0(\Lambda) = 2\nu^{2\nu-1}$

where

$$
z = 2(g\nu^2 x^{\lambda - 2} \ln x)^{1/2}.
$$
 (E17)

Furthermore, (3.18) is replaced by

$$
F'(x) = dF(x)/dx = \frac{\partial F(x)}{\partial x}
$$

+ $(2v)^{-1}z[1+v(\ln x)^{-1}]x^{-1}\partial F(x)/\partial z$ (F)

$$
+ (2\nu)^{-1}\mathbb{Z}[1 + \nu(\ln x)^{-1}]x^{-1}\partial P(x)/\partial z, \quad \text{(E18)}
$$
 and

$$
F'(x) \sim \Gamma(\nu) \left(\frac{1}{2}z\right)^{-\nu+1} I_{\nu-1}(z).
$$

$$
\Gamma(n+1+\nu)n!\]^{-1}
$$

×[1+O((ln x)^{-1})]. (E15)
$$
\int dz z^{-1}[I_{\nu-1}(z)]^{-2} = -K_{\nu-1}(z)/I_{\nu-1}(z), \quad (E20)
$$

$$
B_0(\Lambda) = 2\nu^{2\nu-1} [\Gamma(\nu)]^{-2} (g \ln \Lambda)^{\nu} K_{\nu-1}(\Omega) / I_{\nu-1}(\Omega), \quad (E21)
$$

where

$$
\Omega = 2(g\nu^2\Lambda^{\lambda-2}\ln\Lambda)^{1/2},\tag{E22}
$$

and that as $\Lambda \rightarrow \infty$.

$$
B_n(\Lambda) = (-1)^{n+1} \nu^{2\nu} \left[\Gamma(1+\nu) \right]^{-1}
$$

$$
\times \Gamma(1-\nu) C_n^{\nu} (g \ln \Lambda)^{\nu} + o(1), \quad (E23)
$$

(E19) where C_n^{ν} is as before the binomial coefficient.

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Charged-Pion Photoproduction from Deuterium with Polarized Bremsstrahlung*

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Measurements have been made on the ratio of pion-production cross sections at right angles to and along the photon electric-field vector. The positive and negative pions were first momentum-analyzed and counted by means of a counter telescope. Data have been taken at 45, 90, and 135° in the cm. system, and at proton energies of 225, 330, and 450 MeV. A comparison of the data is made with the dispersion-relation calculation of McKinley.

I. INTRODUCTION

THE photoproduction of positive and negative pions
from deuterium has been extensively studied in
the energy region from threshold to 500 MeV.¹ In all HE photoproduction of positive and negative pions from deuterium has been extensively studied in previous experiments, either the total cross sections or the angular distributions were observed. The present experiment concerns the asymmetry of the pions photoproduced by polarized gamma rays. The positive pion production from polarized gamma rays has been studied in this laboratory² and the present experiment is an extension to the study of negative pion production from deuterium. Measurements were made at photon energies of 225, 330, and 450 MeV.

The production asymmetry *A* is defined as $(\sigma_1 - \sigma_{II})$ / $(\sigma_1+\sigma_{11})$, where σ_1 and σ_{11} refer to the meson-production cross section perpendicular and parallel to the plane of polarization of the photon. The measurement of *A* for positive pion production from hydrogen has shown some disagreement with the dispersion-relation calculations,

and no reasonable set of pion-nucleon phase shifts can make those calculations compatible with the observed angular behaviors of the asymmetry. Moreover, the introduction of $\gamma \pi \rho$ coupling does not improve the agreement appreciably. The present experiment shows the same discrepancy between the theory and the measured values. The measurements were made at energies sufficiently remote from the pion-production threshold that the final-state interaction can reasonably be neglected. For photon energies between 200 and 500 MeV, the analysis of meson production from deuterium in terms of free-nucleon cross sections has been demonstrated to be satisfactory.

II. EXPERIMENTAL METHOD

A polarized bremsstrahlung beam, developed by Taylor and Mozley,² was produced by placing a thin (0.003-in.) aluminum foil at the end of the Stanford Mark III linear accelerator. A beam of electrons striking the foil produced bremsstrahlung polarized perpendicular to the plane of emission. The polarization is a function of the angle which the photon makes with the initial direction of the electron, reaching a maximum at an angle of mc^2/E_0 , where E_0 is the initial electron energy. We have calculated the polarization

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Force Office of Scientific Research.
¹ D. H. White, R. M. Schectman, and B. M. Chasan, Phys.
Rev. 120, 644 (1960), and references therein.
² R. E. Taylor and R. F. Mozley, Phys. Rev. 117, 835 (1960);
R. C. Smith and R.

according to the work of May,³ and after multiplescattering correction and aperture fold, a polarization of about 20% was obtained. The electron energy was not rigidly controlled and could vary by an amount between $\frac{3}{4}\%$ and 2% , and the photon polarization varied from run to run. However, during any one run, the photon polarization remained constant, as was verified² by noting the size and location of the electronbeam spot. By using the method of Taylor and Mozley,² the effect of multiple scattering and electron divergence was taken into account by measuring the undeflected beam size at the position of the collimator. During the experiment, the electrons were deflected by a magnetic field after passing through the radiator, and measured by a secondary emission monitor (see Fig. 1).

The equipment for the production and detection of pions consisted essentially of a liquid-deuterium target, spectrometer, and associated electronic circuits. This apparatus was described earlier.² Special care was taken to shield the photomultiplier tubes from the magnetic field of the spectrometer. The liquid-target system contained two separate 10-in.-long by $1\frac{1}{2}$ -in.-diam cylindrical targets suspended one above the other, and fed by condensers immersed in a liquid-hydrogen reservoir. The targets were made of 0.002-in. stainless steel and plated with 0.0005-in. nickel. The two identical targets were mounted on a bellows system in a vacuum chamber, so that either one of the two targets could be placed in the beam line. The photoproduced pions were momentum-analyzed and detected by counter telescopes. Three pion telescopes, each consisting of two scintillation counters acting in coincidence, defined three momentum intervals. Each momentum channel had a resolution of 4% , but to improve the statistical accuracy, all the data from three channels were combined to give a resolution of 12% in momentum. Sheets of polyethylene absorbers of sufficient thickness to stop

protons were put in the beam of the momentumanalyzed particles in front of the scintillation counters, and the pulse height from each of the counters was analyzed. The photon beam was monitored by a thin-wall ionization chamber.

The procedure for taking data was as follows: The photon beam was centered about the collimator by sweeping the photon beam across the ion chamber, observing the maximum signal developed in the ion chamber and adjusting auxiliary Helmholtz coils. The main Helmholtz coils were then energized, deflecting the electrons before they struck the radiator so that the radiation passing through the collimator would have an optimum value of polarization.² When the beam was deflected up or down, the collimator would pass a polarized beam, and one group of scalers would record counts proportional to $(N_{i\sigma_{11}}+N_{i\sigma_{1}})$. When the beam was deflected right or left, a second set of scalers would record $(N_{t}\sigma_{1}+N_{\tau}\sigma_{11})$. Thus, by making a complete cycle of the deflected beam, the following ratio was measured:

$R = (N_t \sigma_1 + N_r \sigma_{11}) / (N_t \sigma_{11} + N_r \sigma_1 -).$

Here N_t and N_r refer to the number of photons transverse and parallel to the plane of emission of the photons, so that $P = (N_t - N_r)/(N_t + N_r)$ defines the photon polarization. The asymmetry *A* defined as $(\sigma_1 - \sigma_{II})/(\sigma_1 + \sigma_{II})$ is therefore related to P by $AP=(R-1)/(R+1)$.

Background subtractions were made for pions from the walls of the target, and those produced by gamma rays other than those from the radiator foil. The background from the empty target was of the order of one percent. The background for negative pion data was obtained by substituting the liquid-hydrogen target for the deuterium target, and the count obtained with

TABLE I. The measured asymmetry in percent. The photon energy is in MeV.

Photon energy	Average $photon$ polarization	Measured asymmetry $(\sigma_1 - \sigma_{\text{II}})/(\sigma_1 + \sigma_{\text{II}})$					
		45°	π^+ 90°	135°	45°	π 90°	135°
225 330 450	0.19 0.14 0.15	$18.75 + 5.2$ 39.8 ± 1.5	$12.30 + 4.9$ 43.3 ± 3.4 59.5 ± 5.7	$8.0 + 3.9$ $25.0 + 5.0$ -8.2 ± 19.6	$15.20 + 4.9$ 40.4 ± 1.5	$-2.97 + 4.1$ 41.8 ± 3.3 $40.8 + 5.7$	7.1 ± 3.2 18.4 ± 4.6 $9.6 + 13.1$

M. M. May, Phys. Rev. 84, 265 (1951).

the spectrometer set for negative particles was taken to be the background. This background was generally about 5% . Since the resolution of the scintillation

FIG. 2. (a) A comparison of the theoretical and measured asymmetry of pions from deuterium: positive pions. The cal-culated photon resolution is also shown, (b) A comparison of the theoretical and measured asymmetry of pions from deuterium: negative pions. The calculated photon resolution is also shown.

FIG. 3. (a) The effect of the $\gamma \pi \rho$ coupling on the asymmetry of the positive pion. (b) The effect of the $\gamma \pi \rho$ coupling on the asymmetry of the negative pion.

FIG. 4. A χ -squared plot of all our data as a function of the $\gamma \pi \rho$ coupling parameter Λ in units *ef.*

counters did not, in general, permit the complete separation of pions and electrons on the basis of their energy loss only, the above measurement gave us an estimate of the electron contamination. Since only ratios of meson yields were measured, the error introduced by these backgrounds should not be appreciable.

III. EXPERIMENTAL RESULTS

Table I lists our asymmetry measurements and gives the average value of photon polarization for each point. In Figs. 2(a), (b), we compare the asymmetry *A* measured at different angles with the theoretical values. The curves were drawn from values calculated from the amplitudes given by McKinley.⁴ These amplitudes are similar to those given in Ref. 5, except that in the former, expansion in terms of *1/M* was not made. The contribution of the ρ meson was included in the amplitudes and the $\gamma \pi \rho$ coupling constant Λ was treated as a parameter. The phase shift used in the calculation was the set labeled x in Ref. 4. In this comparison, no energy or angular fold of the theoretical values was

made. The photon resolution for the experimental results is shown in Fig. 2. These values were obtained by folding in the momentum resolution of the spectrometer and the internal momentum distribution of the nucleons. The errors shown are of a statistical nature only.

Kinematic calculations show that pion-pair contamination could come from the very peak end of the bremsstrahlung, and since the pair cross section is negligible below 500 MeV, our data for the two lower energies should be free from this error. Even at the 450-MeV points, the pair cross section is not expected to contribute to the measured asymmetry because the photons at the peak end of the bremsstrahlung have no polarization.

The disagreement between the measured asymmetry and the theoretical values is most pronounced at energies away from the resonance. The inclusion of the $\gamma\pi\rho$ contribution does not improve the comparison to any significant extent.

Figures $3(a)$ and $3(b)$, show the effects of the *p*-meson contribution. The effect of the phase shifts was also investigated, but no definite conclusions could be reached. Within the uncertainties of the present knowledge of the phase shifts, a variation of up to 40% of the asymmetry can be obtained, but the overall angular fit is seldom improved.

Figure 4 shows a χ -squared fit of all the data to the theoretical values of the asymmetry as a function of Λ . A minimum is found at $\Lambda = +0.1$, but the x-squared value itself is 76, with 16 points giving a negligible probability for the distribution to be the correct one.

It is interesting that our backward angle data (135°) gives evidence for a large negative Λ as found by Robinson et al.,⁶ although another attempt⁷ to determine Λ from the $-\prime +$ ratio for photoproduction from deuterium near threshold gave $A = +0.1 \pm 0.3$ in agreement with our result.

In summary, our results indicate small coupling at the $\gamma\pi\rho$ vertex, but the over-all agreement with theory is poor.

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⁴ J. M. McKinley, Tech. Report No. 38, Physics Department, University of Illinois (unpublished). J. M. McKinley, Rev. Mod. Phys. 35, 788 (1963).
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⁶ G. F. Chew, M. L. Goldberger, E. F. Low, and Y. N

⁶ C. S. Robinson, P. M. Baum, L. Crieger, and J. M. McKinley, Phys. Rev. Letters 9, 349 (1962).

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